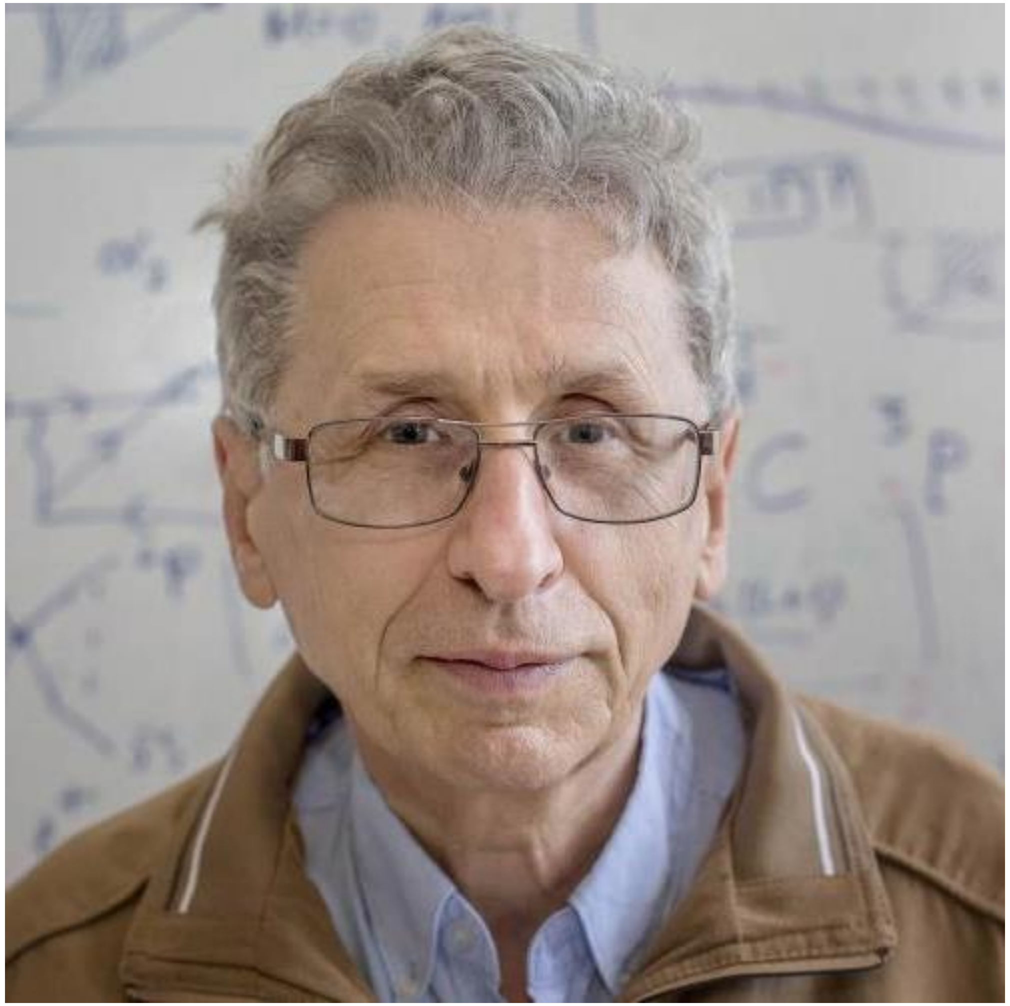
Spin & Gravity

a quantum story

Pavel Fadeev

Auckland 04.02.2020



Dmitry Budker Victor Flambaum

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Auckland 04.02.2020

Newtonian gravity in quantum mechanics

Observation of Gravitationally Induced Quantum Interference

R. Colella and A. W. Overhauser and S. A. Werner

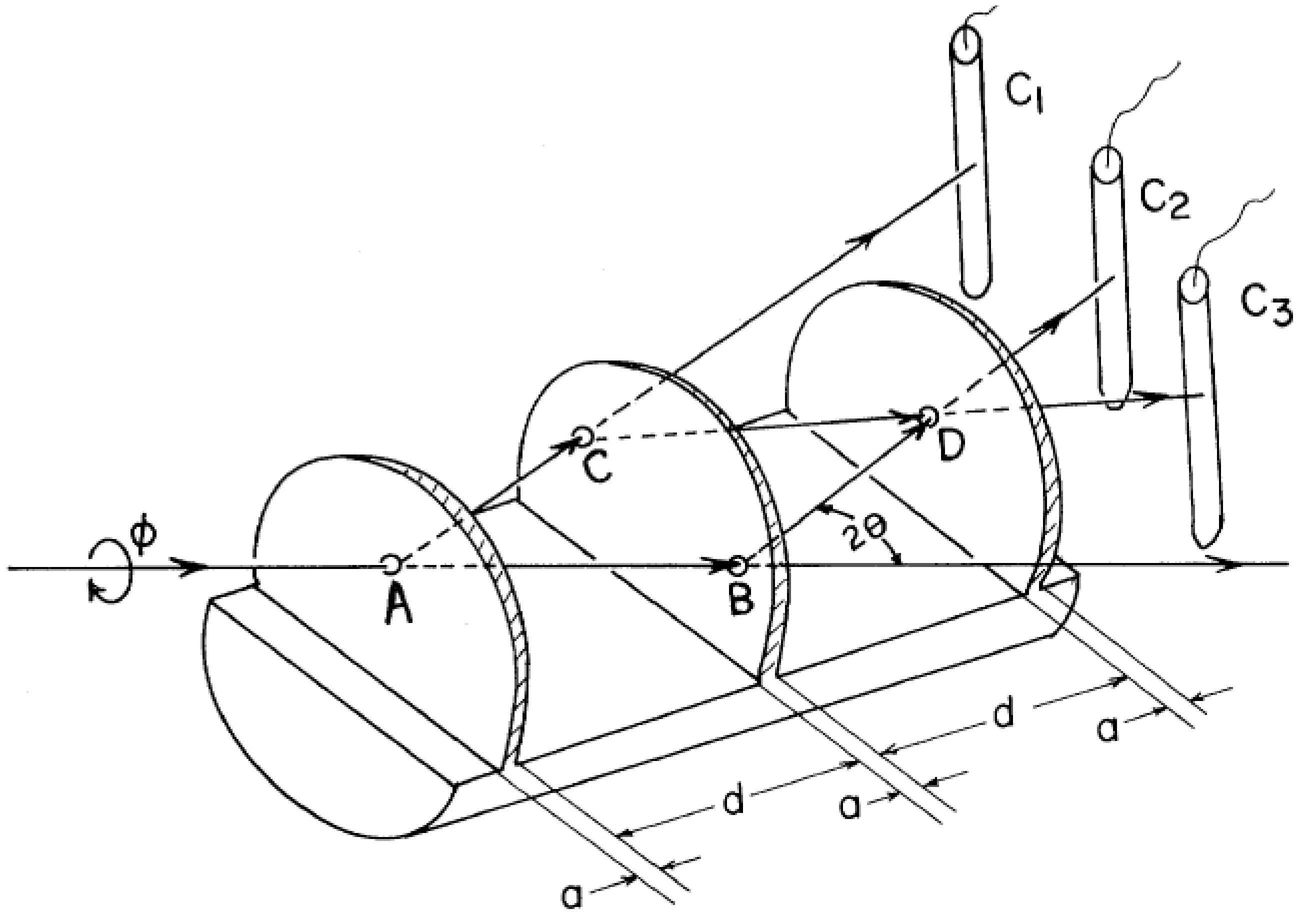


FIG. 1. Schematic diagram of the neutron interferometer and ^{3}He detectors used in this experiment.

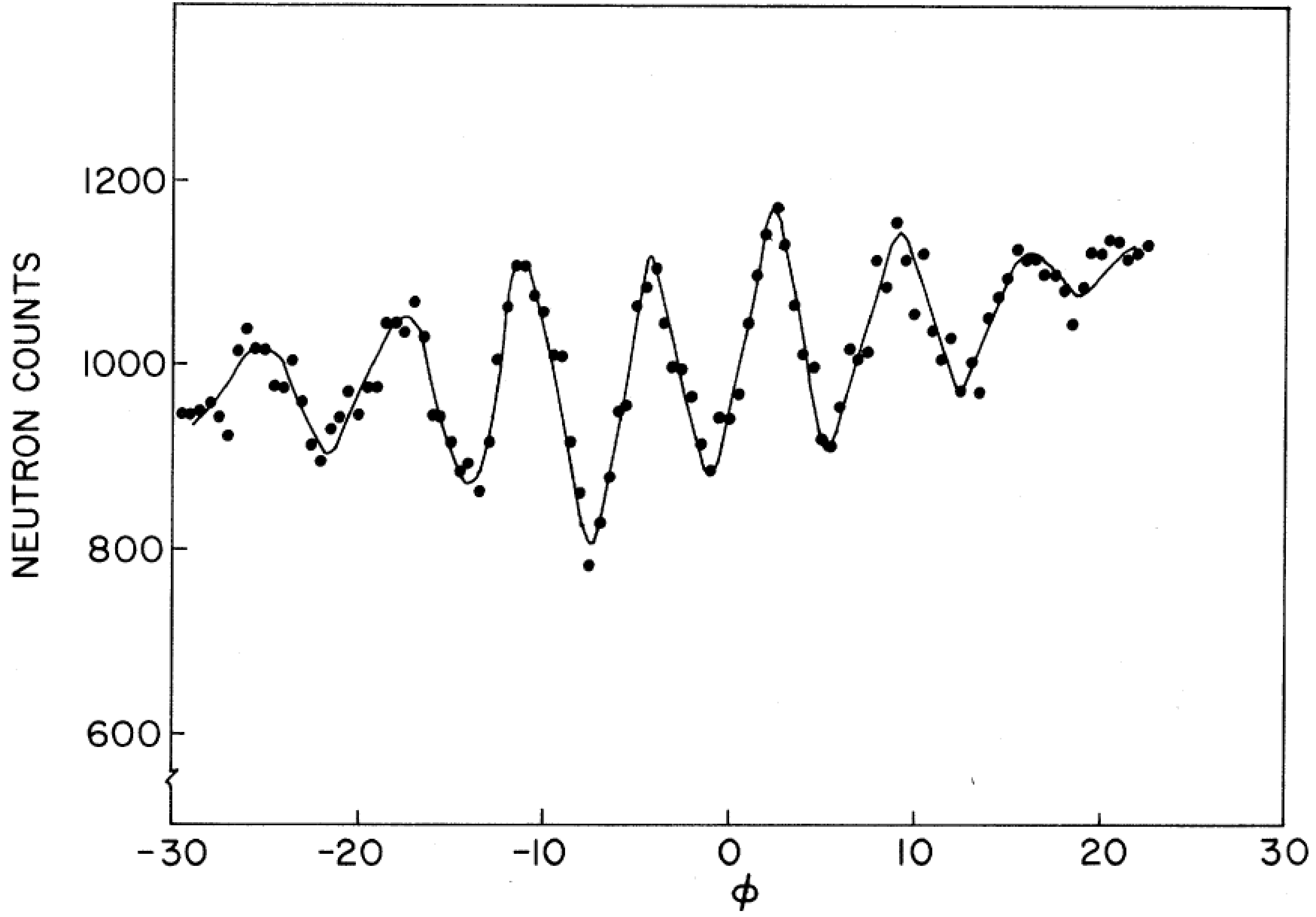
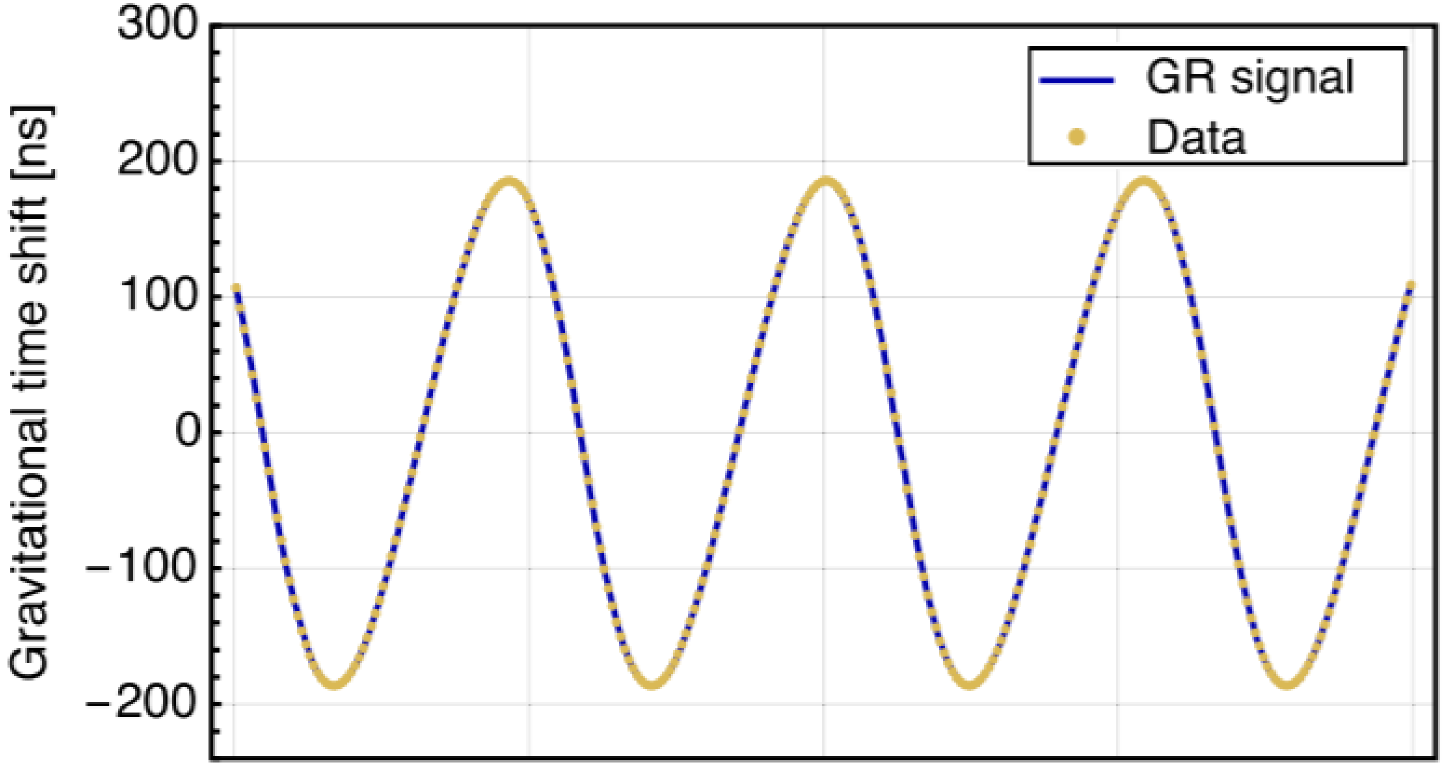
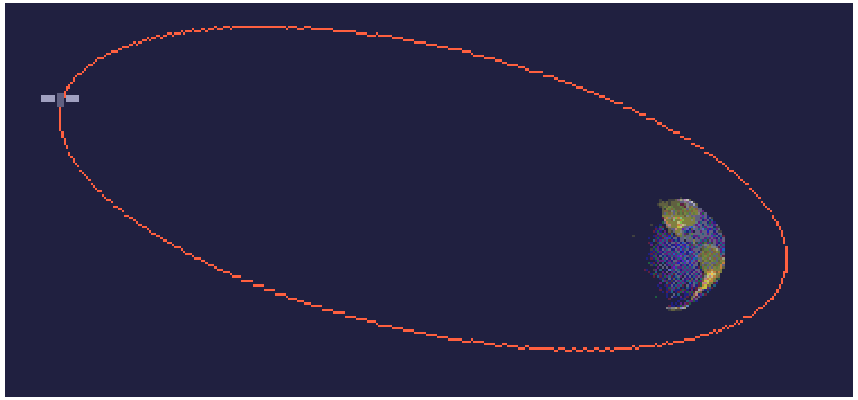


FIG. 2. The difference count, $I_2 - I_3$, as a function of interferometer rotation angle φ.

General relativity in quantum mechanics



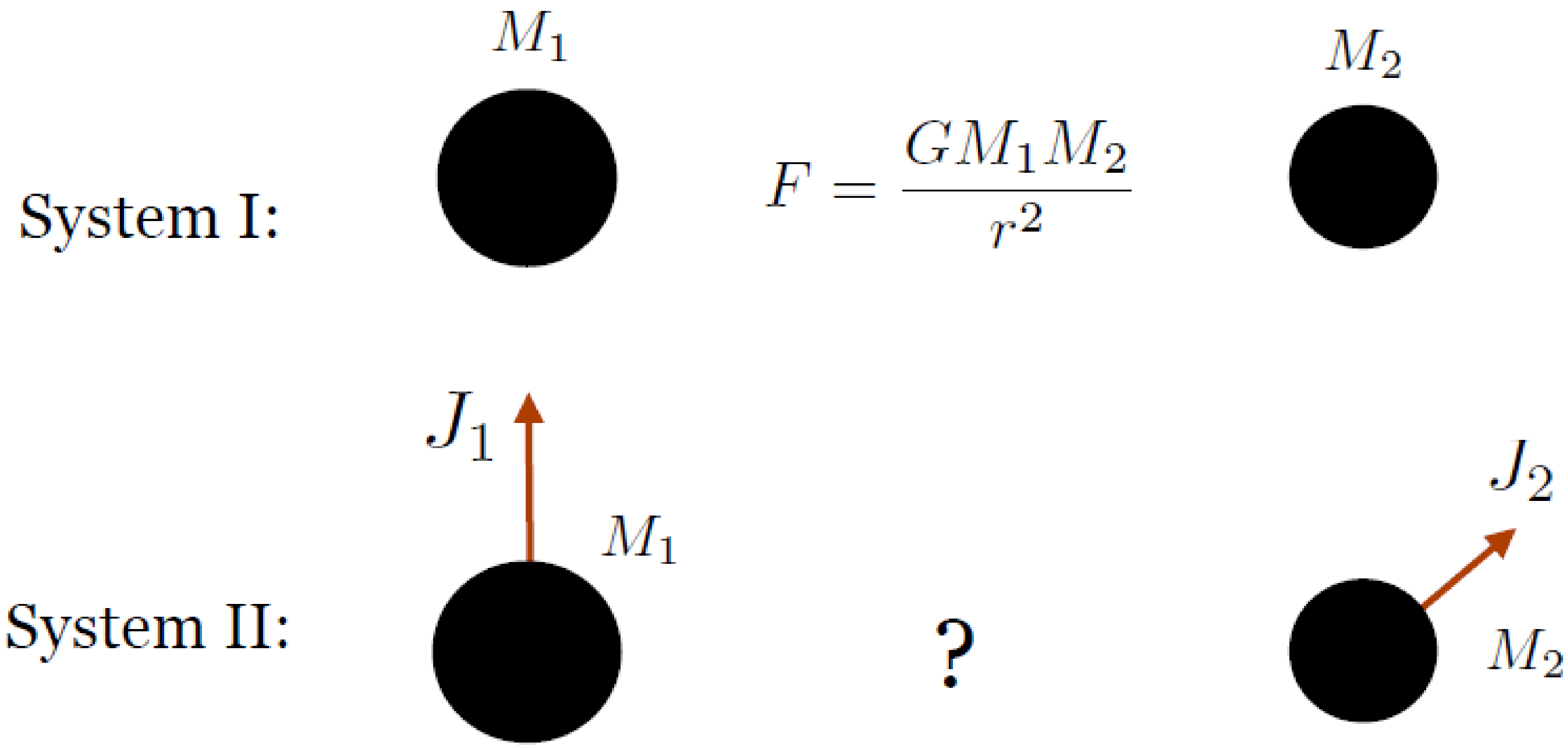


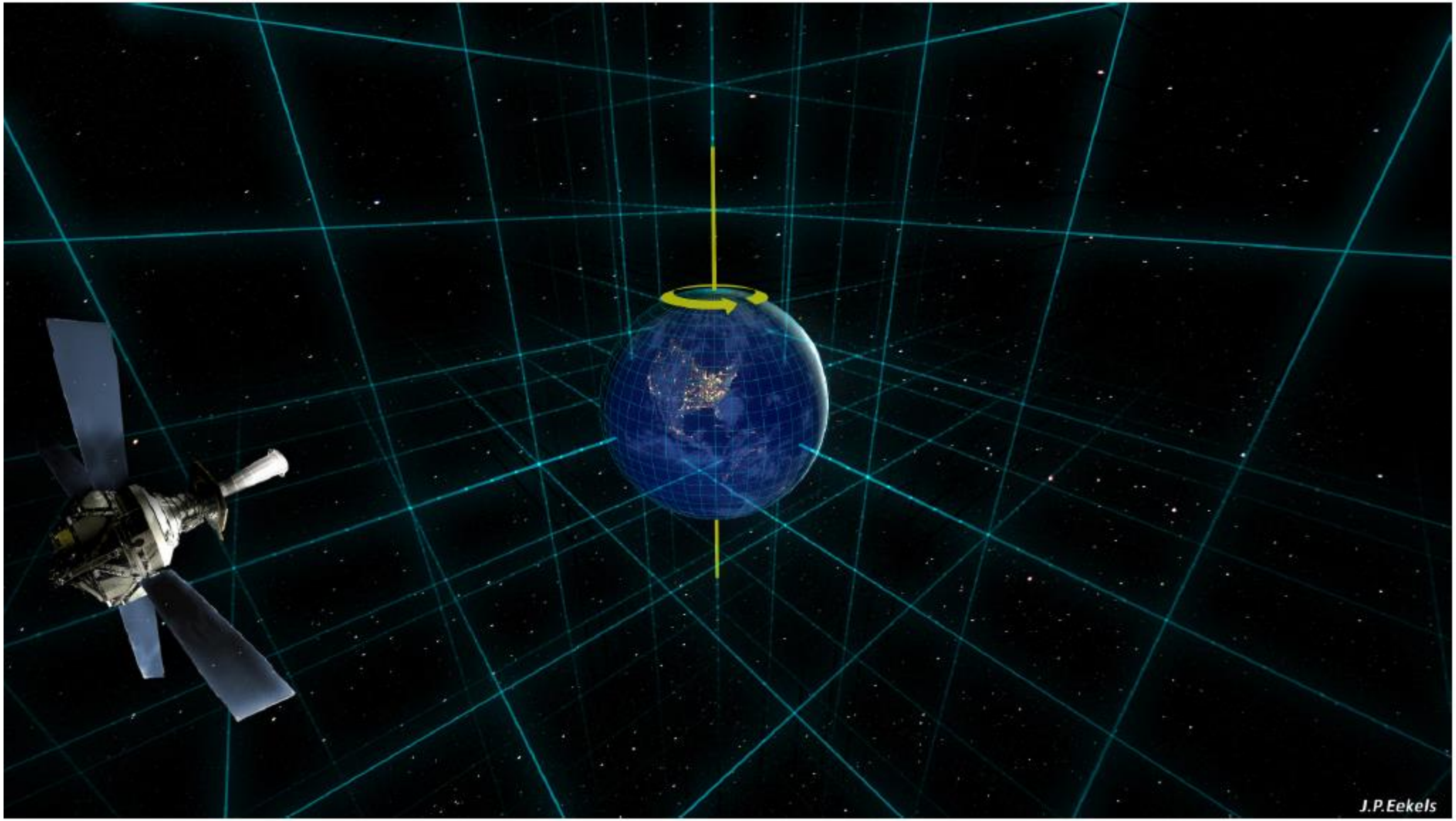
GR prediction, clock data and residuals are shown for 2 days from 31 March 2016.

Delva, P.; Puchades, N.; Schönemann, E.; Dilssner, F.; Courde, C.; Bertone, S.; Prieto-Cerdeira, R. A new test of gravitational redshift using Galileo satellites: The GREAT experiment. *Comptes Rendus Phys.* **2019**, *20*, 175–182.

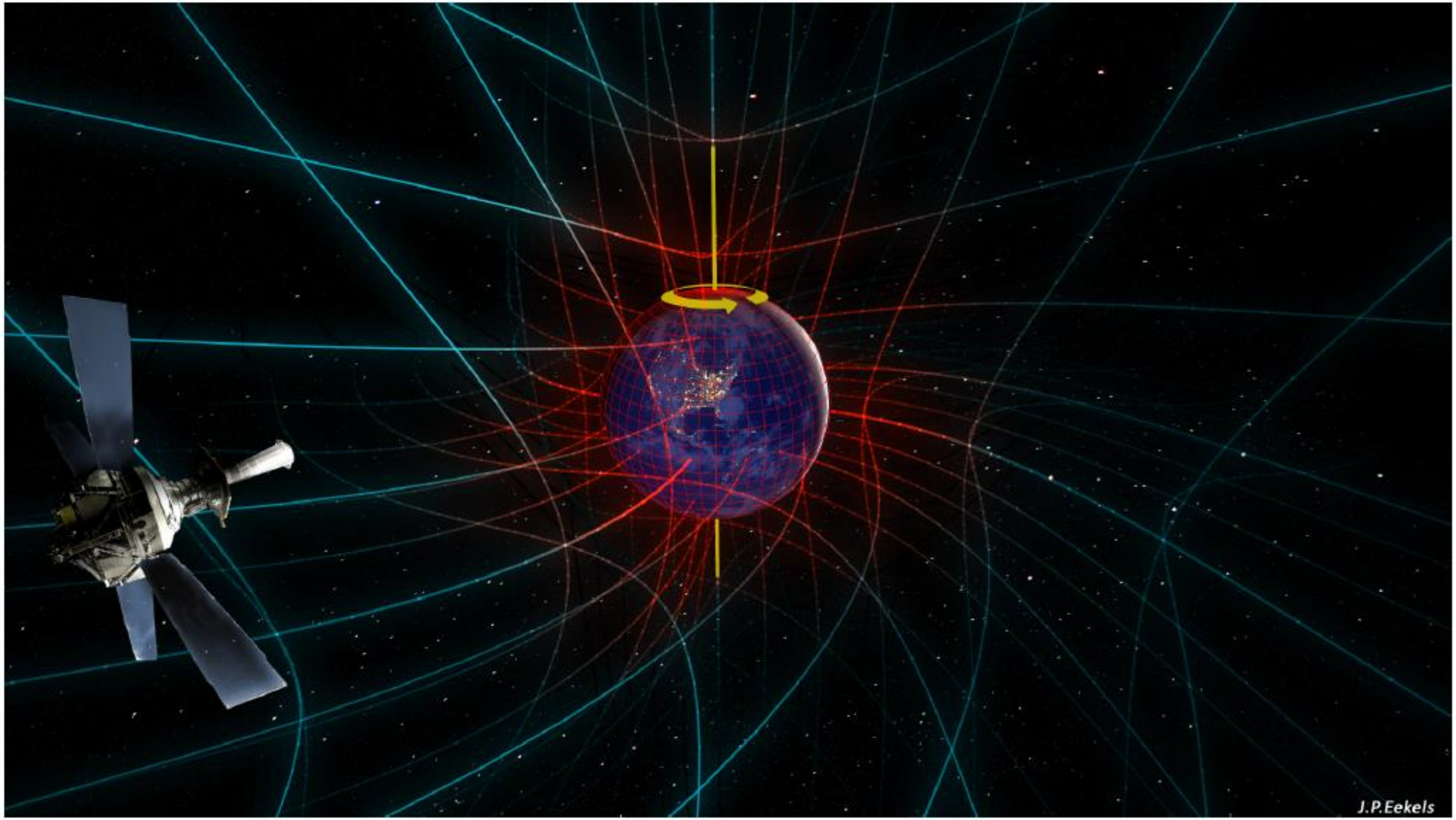
Newtonian Gravity & Spin

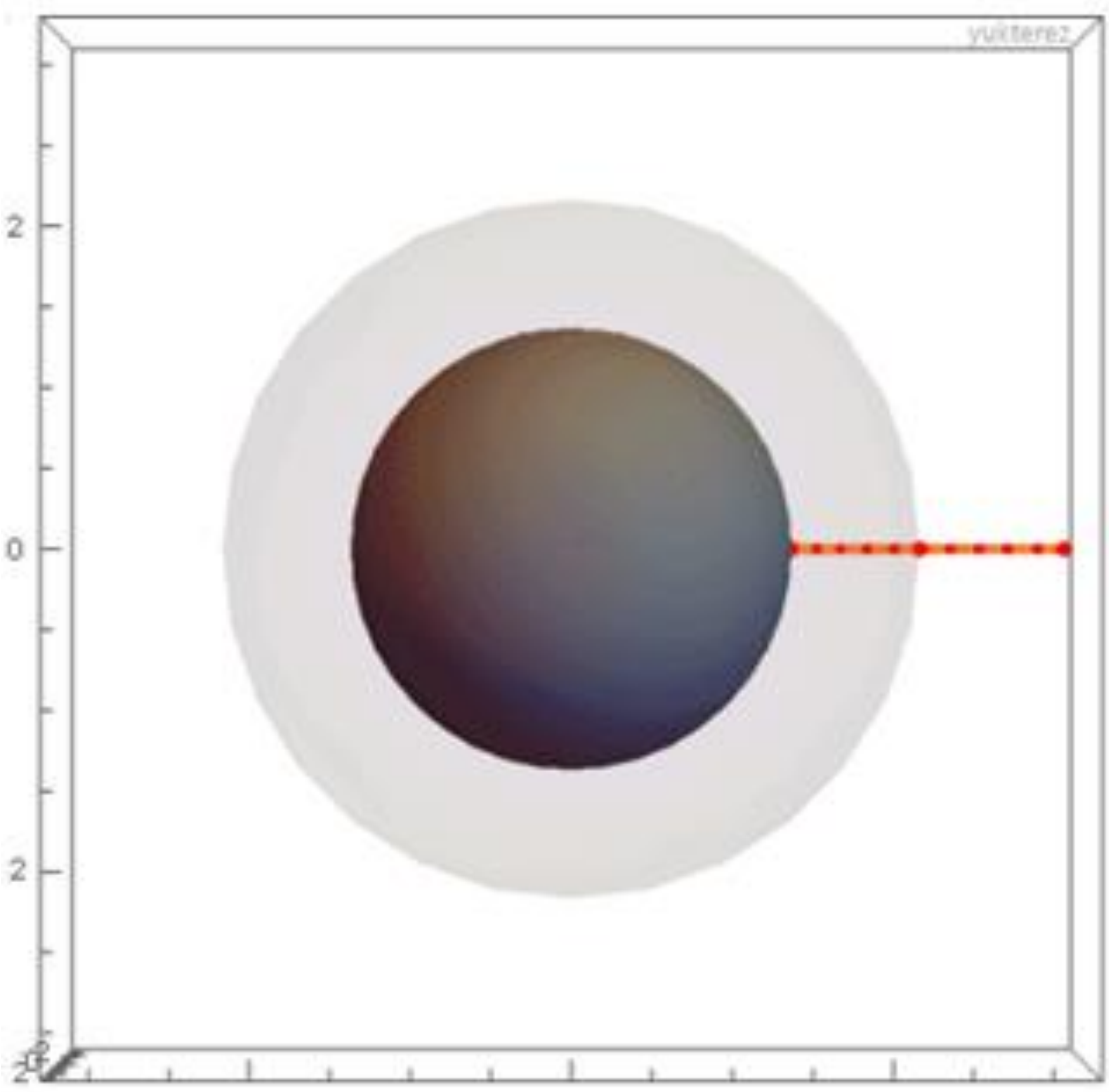
Newtonian Gravity & Spin



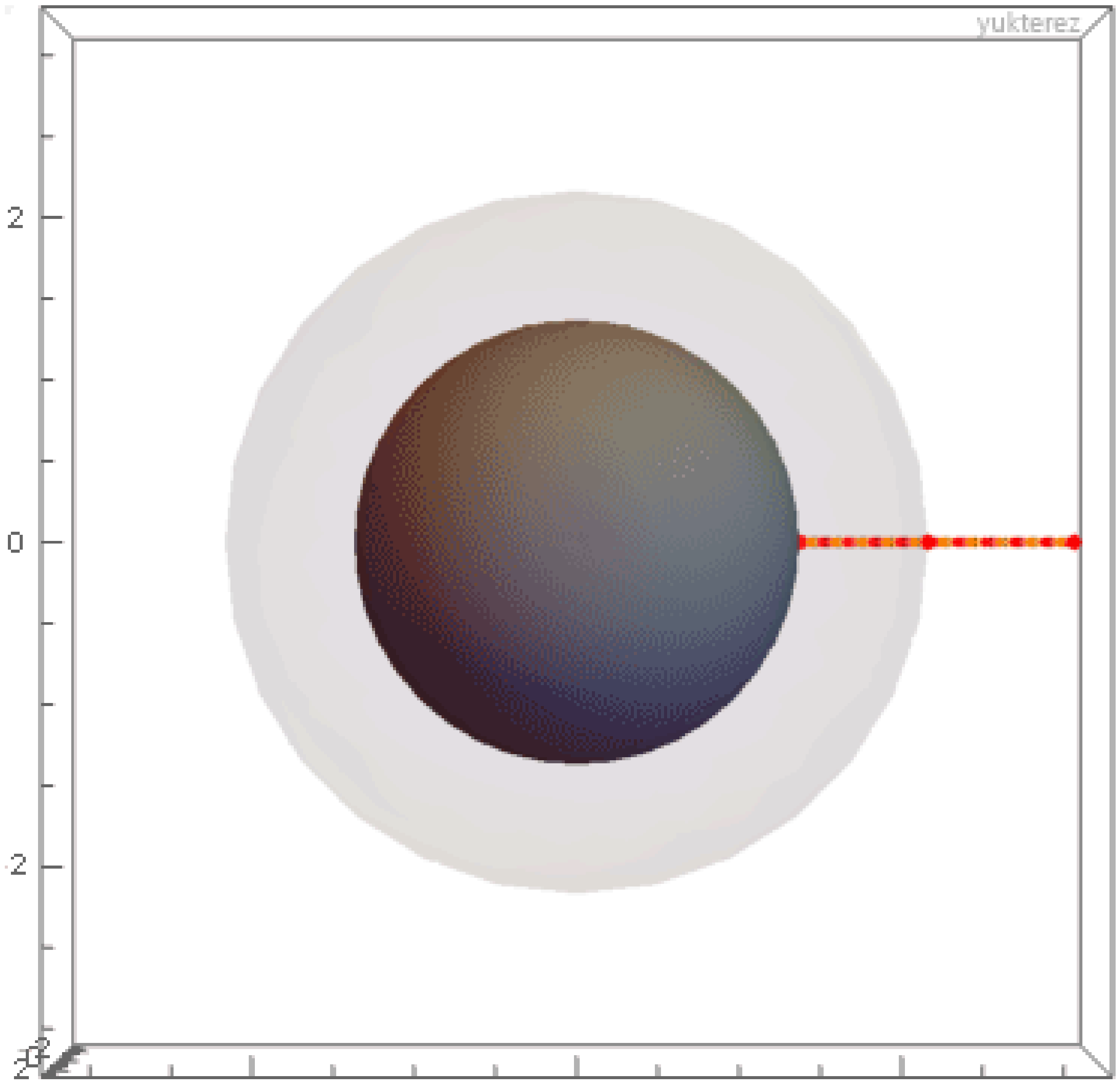

J.P.Eekels

General relativity & Spin

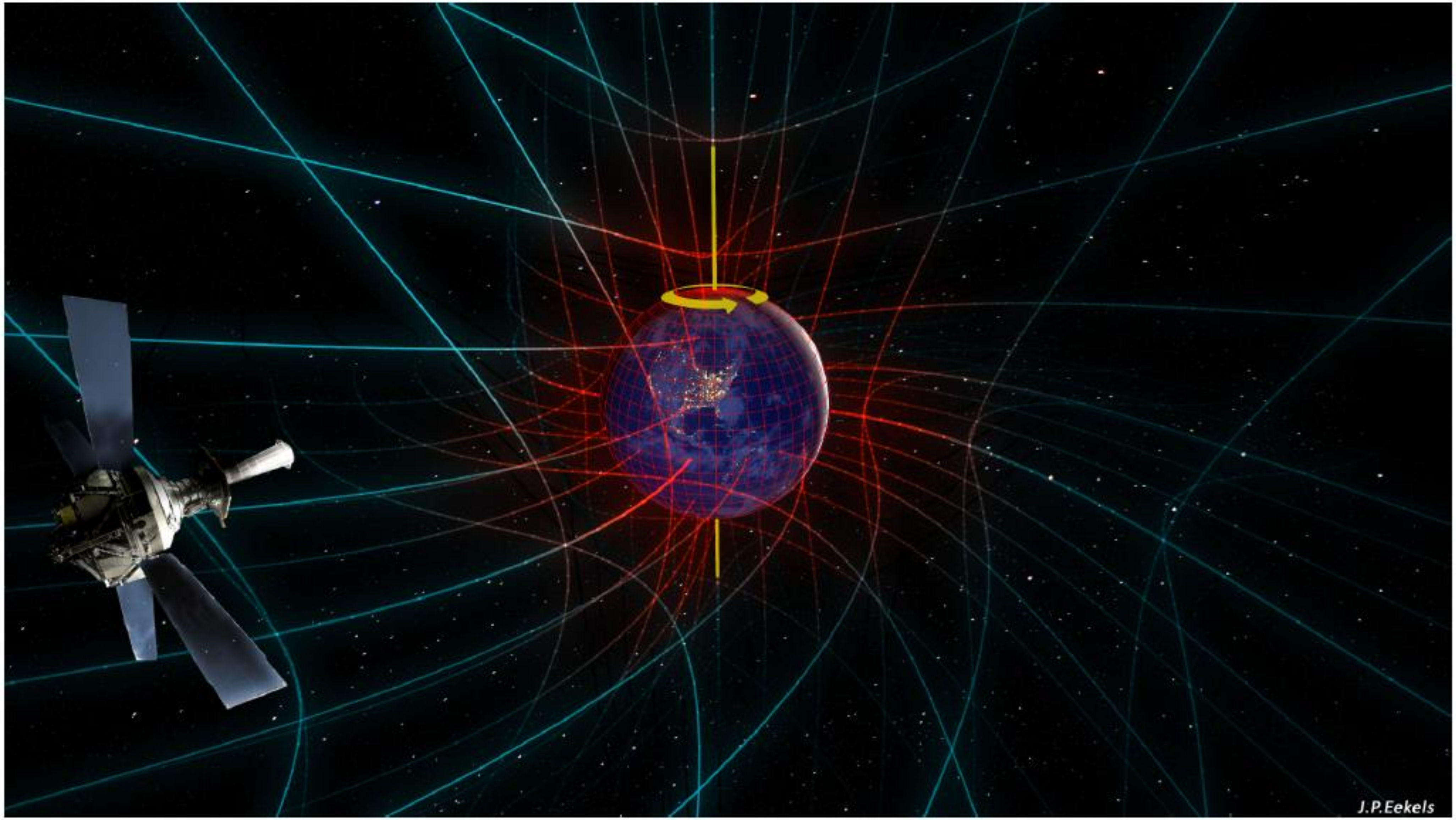

J.P.Eekels



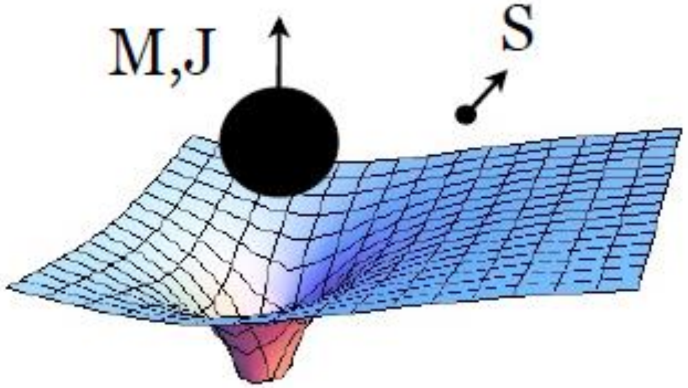
Frame dragging, Wiki. Simon Tyran, Vienna (2017)



Frame dragging, Wiki. Simon Tyran, Vienna (2017)


J.P.Eekels

General relativity & Spin



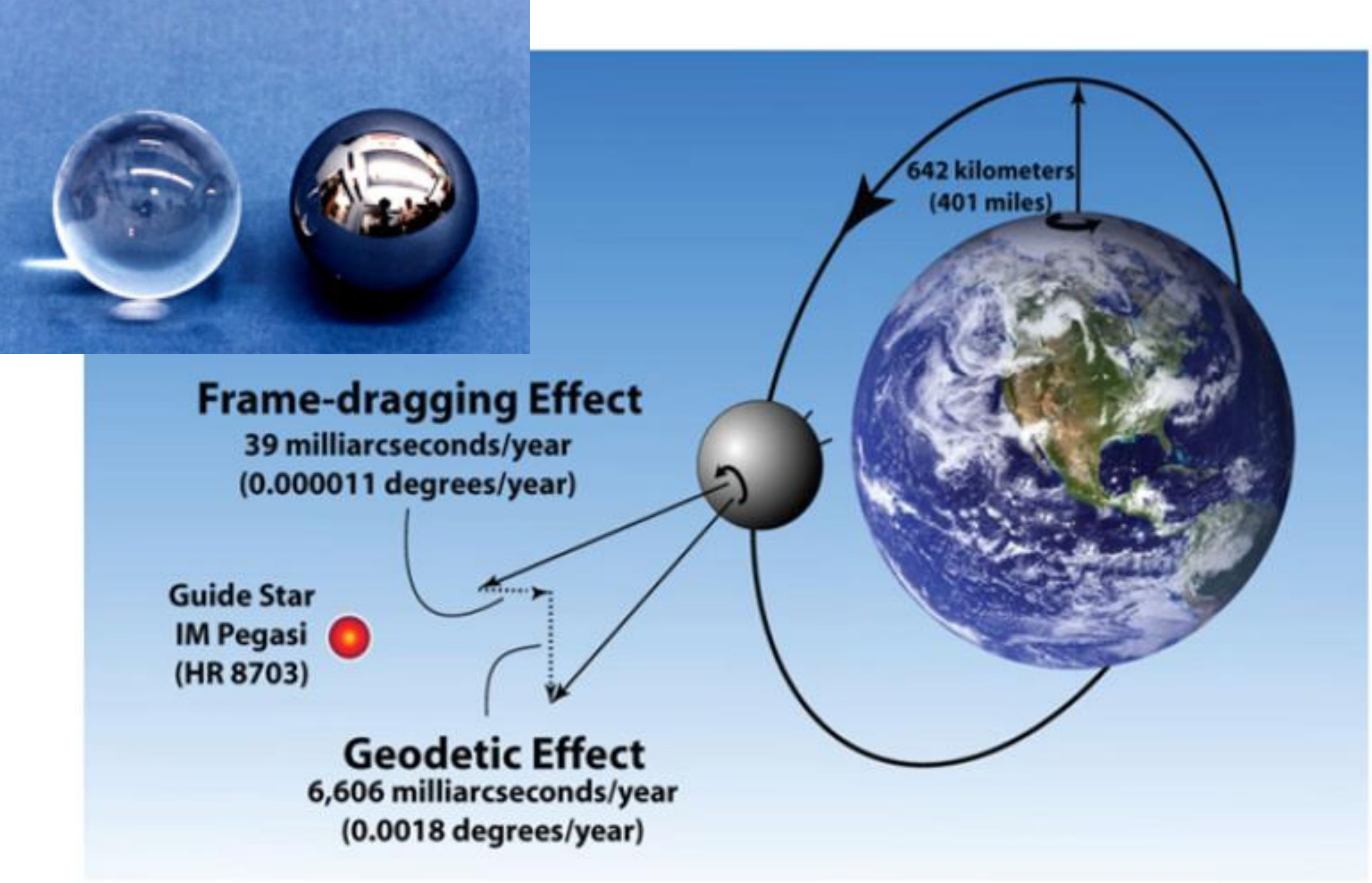
(Figure credit: L. Barack)

- Spin is forced to **precess**: $\partial_t \vec{S} = \vec{S} \times \vec{\Omega}_{\rm p}$
- Precession frequency [Schiff (1960)]:

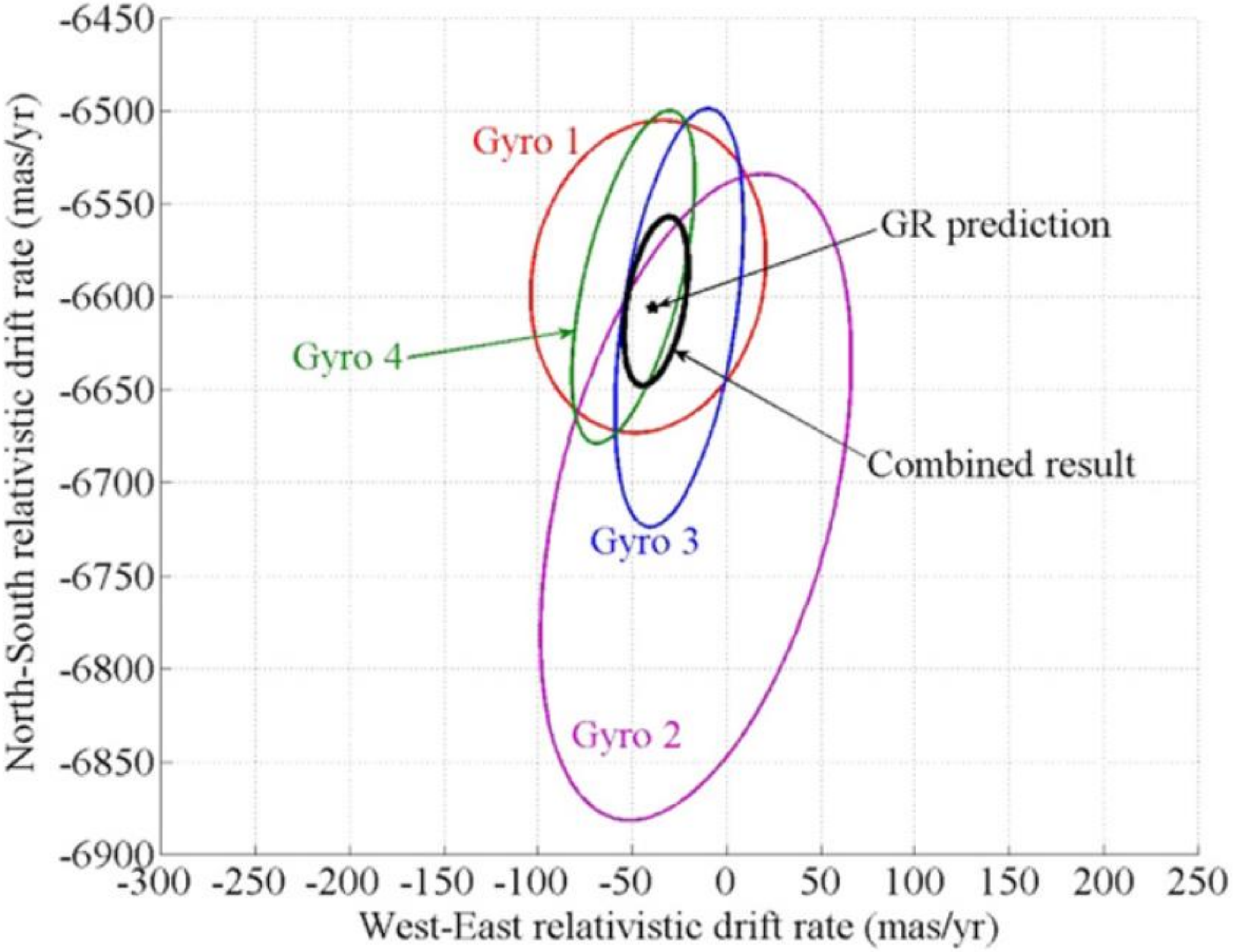
$$\vec{\Omega}_{\rm p} = \frac{3GM}{2c^2r^3}(\vec{r} \times \vec{v}) + \frac{GJ}{c^2r^3}\left[3(\hat{J}\cdot\hat{r}) - \hat{J}\right]$$

↓ Mass-spin coupling. This is the **geodetic effect.**

↓ Spin-spin coupling. This is **frame-dragging.**



C.W. F. Everitt et. al., PRL **106**, 221101 (2011)

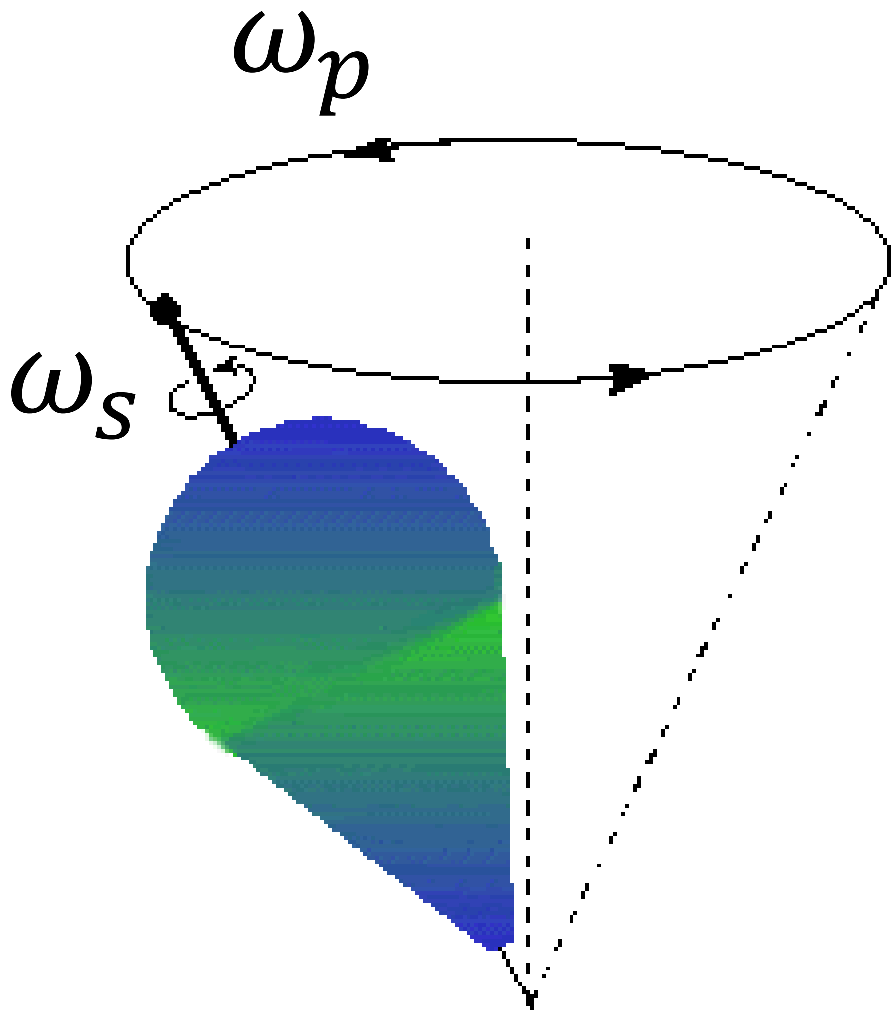


C.W. F. Everitt et. al., PRL **106**, 221101 (2011)

General relativity & internal Spin

$$H = \cdots + \hbar\boldsymbol{\Omega} \cdot (\mathbf{l} + \mathbf{S})$$

M. Pavlichenkov, *EPL* **85,** 40008 (2009).
Giorgio Papini, Gen Relativ Gravit 40, 1117 (2008).



ω_p
ω_s

Threshold in magnetic field to realize precession of a ferromagnetic needle (with moment of inertia I):

$$L \ll S \qquad \Rightarrow \qquad I\Omega \ll N\hbar \ ,$$

$$\Omega \ll \Omega^* = \frac{N\hbar}{I} \ ,$$

$$B \ll B^* = \frac{\hbar\Omega^*}{g\mu_B} \ .$$

$$\Omega^* = \frac{N\hbar}{I}$$

Taking 10^{20} spins and $r = 1\text{mm}$

$$\Omega^* = 10^{-3}\,\frac{1}{s}$$

$$B^* = 10^{-14}\ \text{T}$$

What is a possible magnetic sensitivity? Take a magnetic sensor that observes:

$$B_{obs} = B_{true} + \langle \delta B \rangle$$

Mean magnetostatic energy:

$$E = \frac{V}{2\mu_B} < B_{obs}^2 >$$

$$= \frac{V}{2\mu_B} \left(B_{true}^2 + 2B_{true}\langle\delta B\rangle + \langle\delta B\rangle^2\right)$$

$$= \frac{V}{2\mu_B} \left(B_{true}^2 + \langle\delta B\rangle^2\right)$$

Magnetostatic sensitivity

E

$$\Delta E \;= \frac{V}{2\mu_B}\langle\delta B\rangle^2$$

For measurement Period T:

$$\Delta E \cdot T \;= \frac{V}{2\mu_B}\langle\delta B\rangle^2 \cdot T \geq$$

Magnetostatic sensitivity

E

$$\Delta E = \frac{V}{2\mu_B}\langle \delta B \rangle^2$$

For measurement Period T:

$$\Delta E \cdot T = \frac{V}{2\mu_B}\langle \delta B \rangle^2 \cdot T \geq \hbar$$

$$\frac{V}{2\mu_B}\langle\delta B\rangle^2 \cdot T = \hbar$$

$$\langle\delta B\rangle^2 \cdot T = \frac{2\mu_B}{V}\hbar$$

$$\log\langle\delta B\rangle^2 \cdot T = \log\frac{\hbar}{2\mu_B}\frac{1}{V}$$

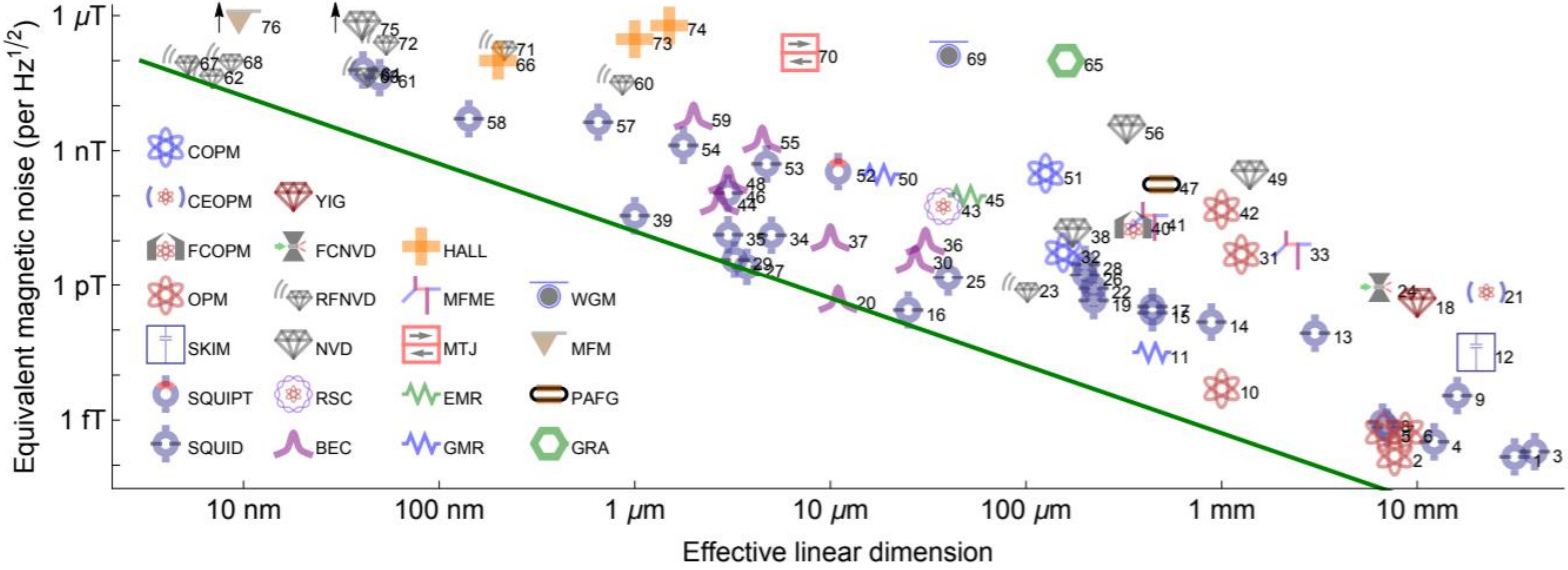
$$= \log\frac{\hbar}{2\mu_B} + \log\frac{1}{V}$$

$$\frac{V}{2\mu_B}\langle\delta B\rangle^2\cdot T=\hbar$$

$$\langle\delta B\rangle^2\cdot T=\frac{2\mu_B}{V}\hbar$$

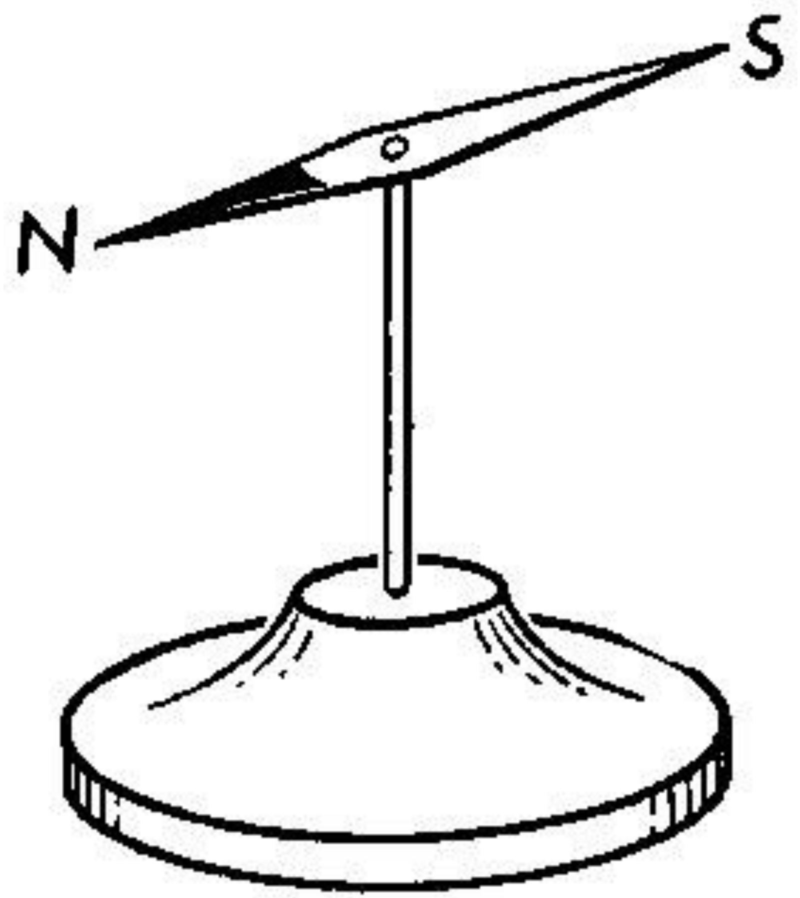
$$\log\langle\delta B\rangle^2\cdot T=\log\frac{\hbar}{2\mu_B}\frac{1}{V}$$

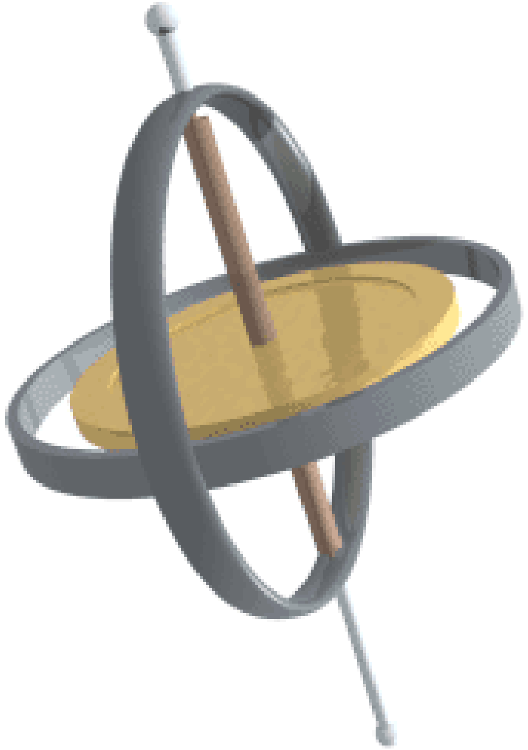
$$=\log\frac{\hbar}{2\mu_B}-\log V$$



M. W. Mitchell and S. P. Alvarez, arXiv:1905.00618 (2019).

Ferromagnetic needle





Two regimes:
precessing and tipping

Precessing regime: $S \gg L$;

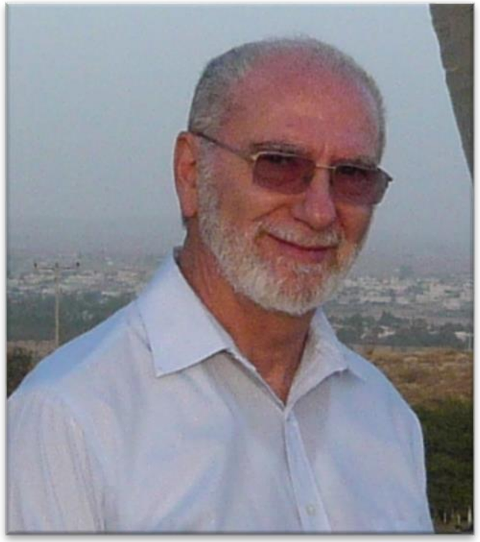
Tipping regime: $L \gg S$.

PRL **116,** 190801 (2016) PHYSICAL REVIEW LETTERS week ending 13 MAY 2016

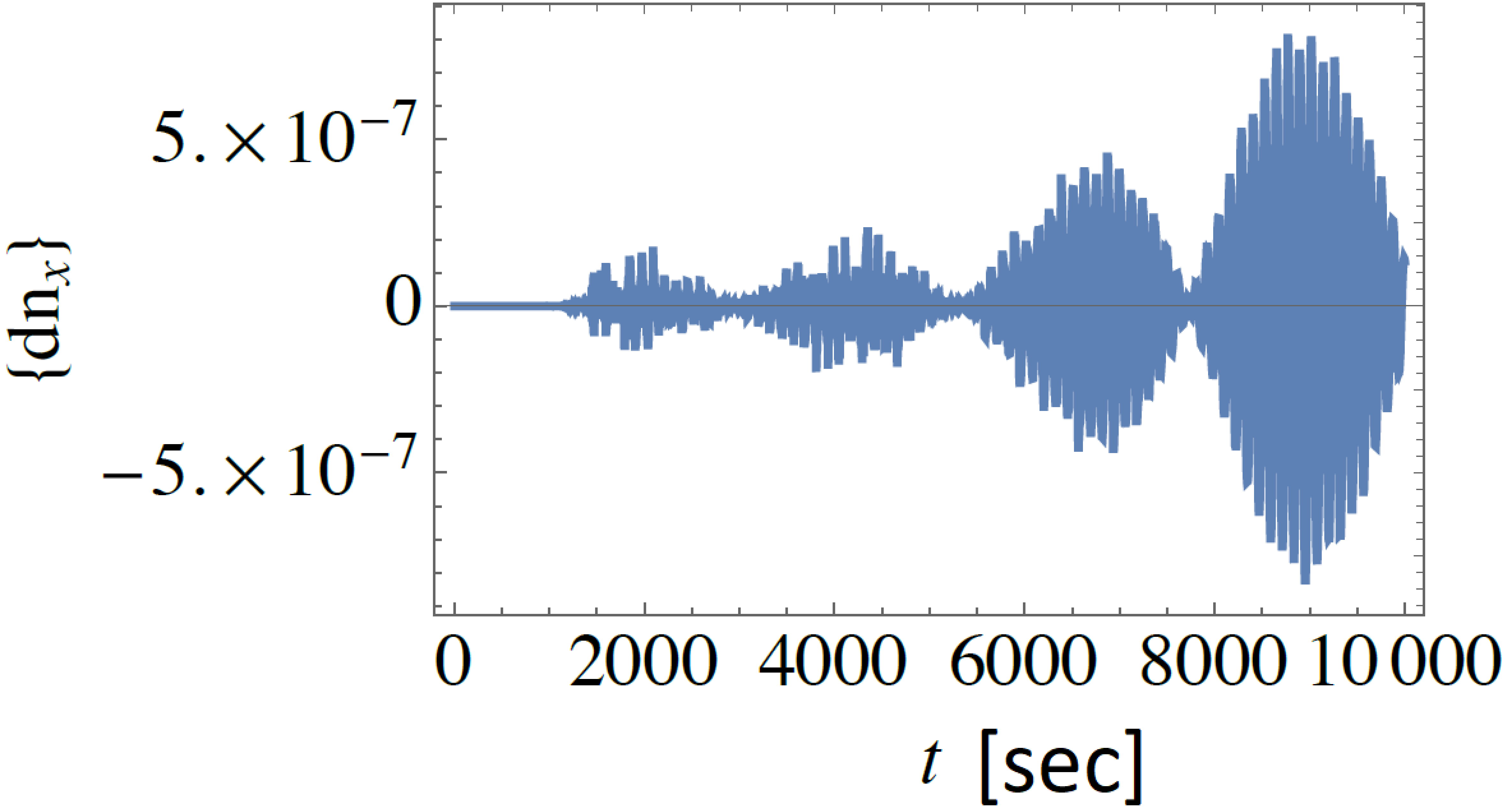
Precessing Ferromagnetic Needle Magnetometer

Derek F. Jackson Kimball,[1] Alexander O. Sushkov,[2] and Dmitry Budker[3,4,5]

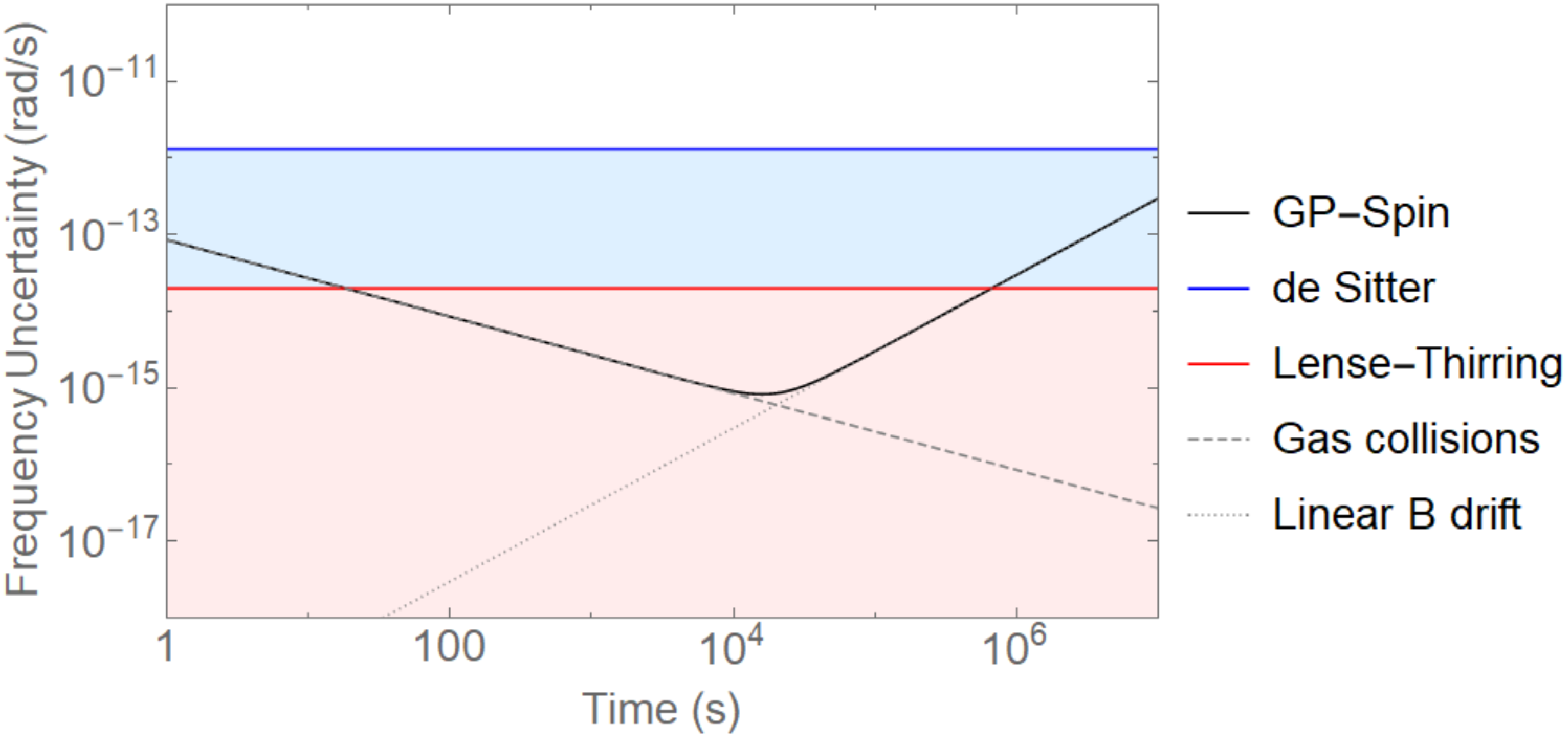
Y. B. Band, Y. Avishai, and A. Shnirman,
Phys. Rev. Lett. **121**, 160801 (2018).



T. Wang, S. Lourette, S. R. Kelley, M. Kayci, Y. B. Band,
D. F. Jackson Kimball, A. Sushkov, and D. Budker,
Physical Review Applied **11**, 044041 (2019).



5. × 10^−7
0
−5. × 10^−7
{dn_x}
0
2000
4000
6000
8000
10 000
t [sec]



Frequency Uncertainty (rad/s)
10^−11
10^−13
10^−15
10^−17
1
100
10^4
10^6
Time (s)
GP–Spin
de Sitter
Lense–Thirring
Gas collisions
Linear B drift

arXiv:1912.10397

Single-Spin Magnetomechanics with Levitated Micromagnets

Jan Gieseler, Aaron Kabcenell, Emma Rosenfeld, J. D. Schaefer, Arthur Safira, Martin J. A. Schuetz, Carlos Gonzalez-Ballestero, Cosimo C. Rusconi, Oriol Romero-Isart, Mikhail D. Lukin

(Submitted on 22 Dec 2019)

"… a path forward to observe precession due to the intrinsic spin angular momentum of the magnet with applications in highly sensitive magnetometry."

arXiv:1912.12252

Ultrahigh mechanical quality factor with Meissner-levitated ferromagnetic microparticles

A. Vinante, P. Falferi, G. Gasbarri, A. Setter, C. Timberlake, H. Ulbricht

(Submitted on 27 Dec 2019)

"We can use our particle to detect ultralow magnetic felds by exploiting the sensitivity of rotational modes to small torques."

Thank you

Pavelfadeev.net

Auckland

04.02.2020

For example, for a cobalt needle with $\ell \approx 10\ \mu$m and $r \approx 1\ \mu$m:

$$\Omega^* \approx 100\ \mathrm{s}^{-1}$$

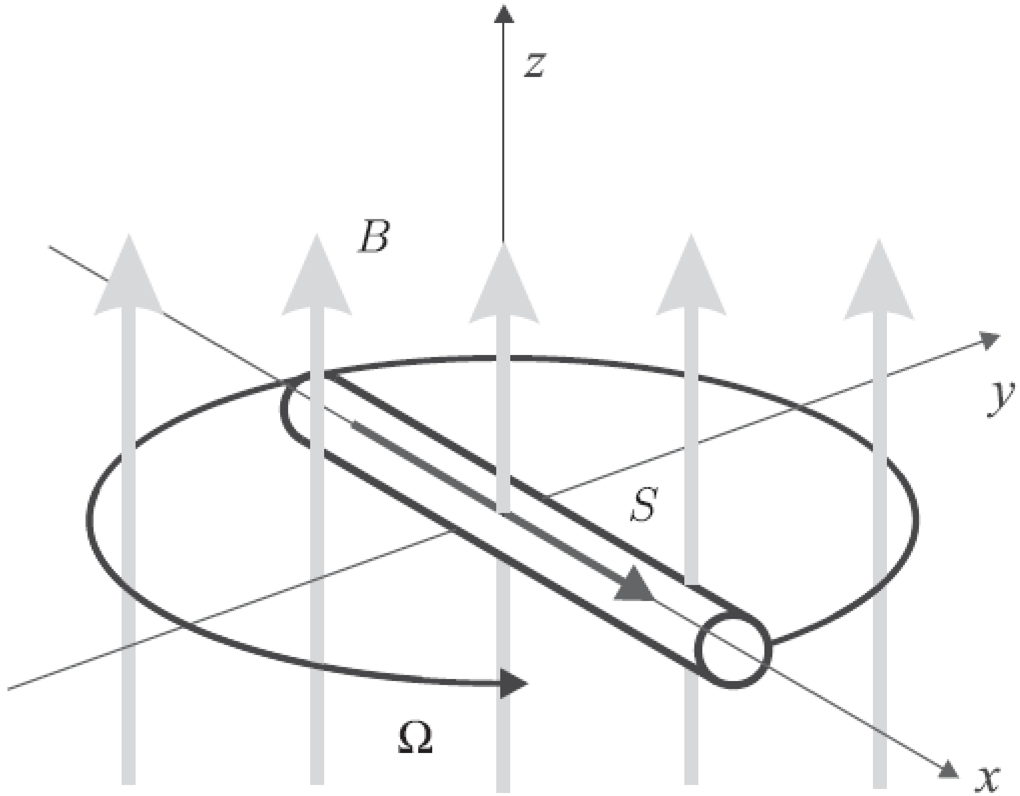
and

$$B^* \approx 10^{-5}\ \mathrm{G}\ ,$$

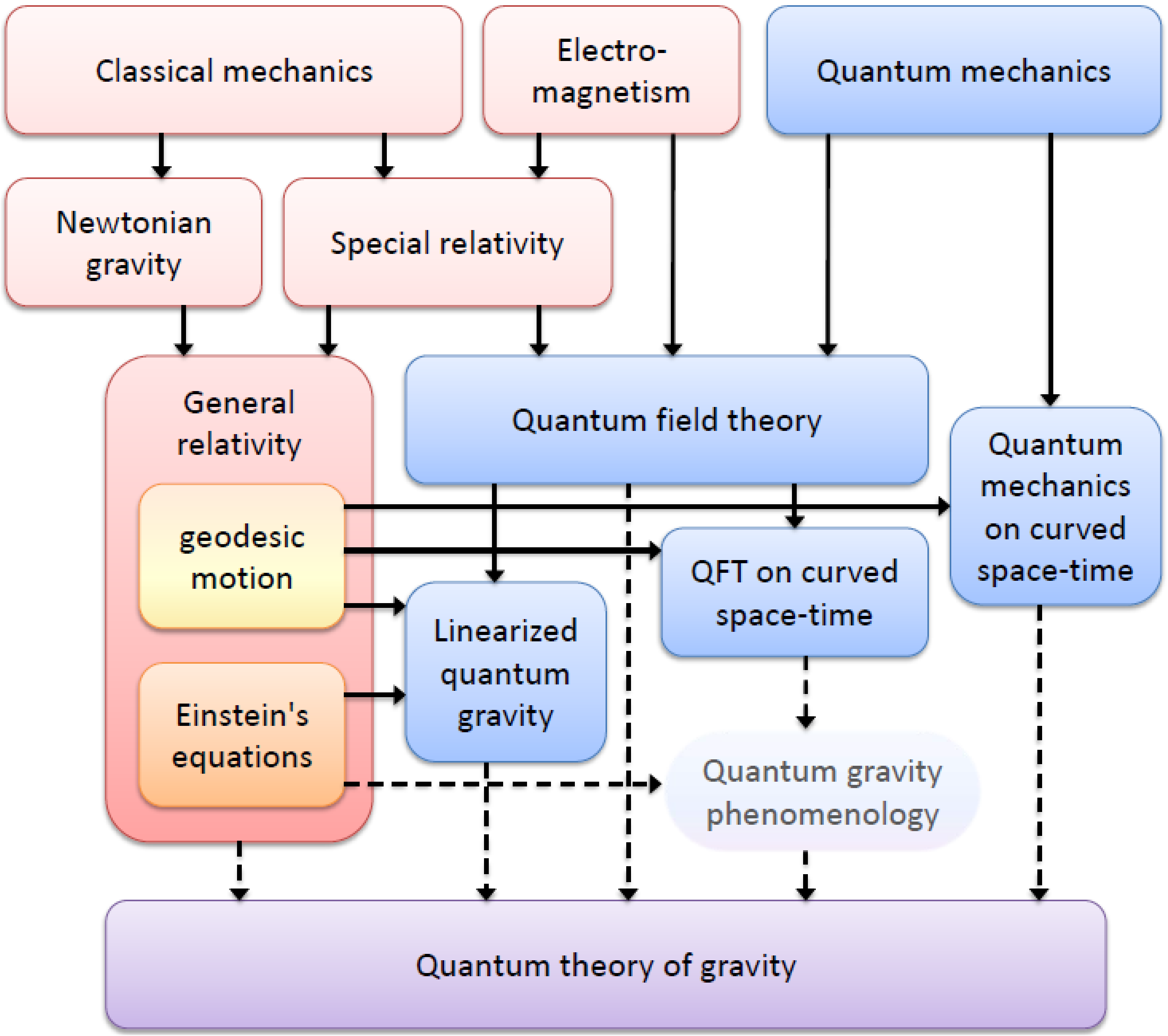
a field value that can be achieved in the laboratory with appropriate shielding.

Characteristic	Notation	Approximate Value
Radius	r	1 mm
Mass density	ρ	8.86 $\mathrm{g/cm^3}$
Mass	$M \approx 4\pi\rho r^3/3$	4×10^{-2} g
Moment of inertia	$I \approx 2Mr^2/5$	$1.6 \times 10^{-4}\ \mathrm{g \cdot cm^2}$
Number of polarized spins	N	4×10^{20}
Gilbert damping constant	α	0.01
Ferromagnetic resonance frequency	ω_0	$10^{11}\ \mathrm{s^{-1}}$
Gyroscopic threshold field	$B^* = N\hbar^2/(g\mu_B I)$	3×10^{-10} G
Gyroscopic threshold frequency	$\Omega^* = N\hbar/I$	$3 \times 10^{-3}\ \mathrm{s^{-1}}$
Operating magnetic field	B	10^{-12} G
Larmor precession frequency	Ω_B	$10^{-5}\ \mathrm{s^{-1}}$
Temperature	T	0.1 K
Background gas density	n	$10^3\ \mathrm{cm^{-3}}$

TABLE I: Proposed characteristics of the orbiting ferromagnetic gyroscope (FG) system for a proposed measurement of general-relativistic spin precession. Assumes a fully magnetized cobalt sphere.

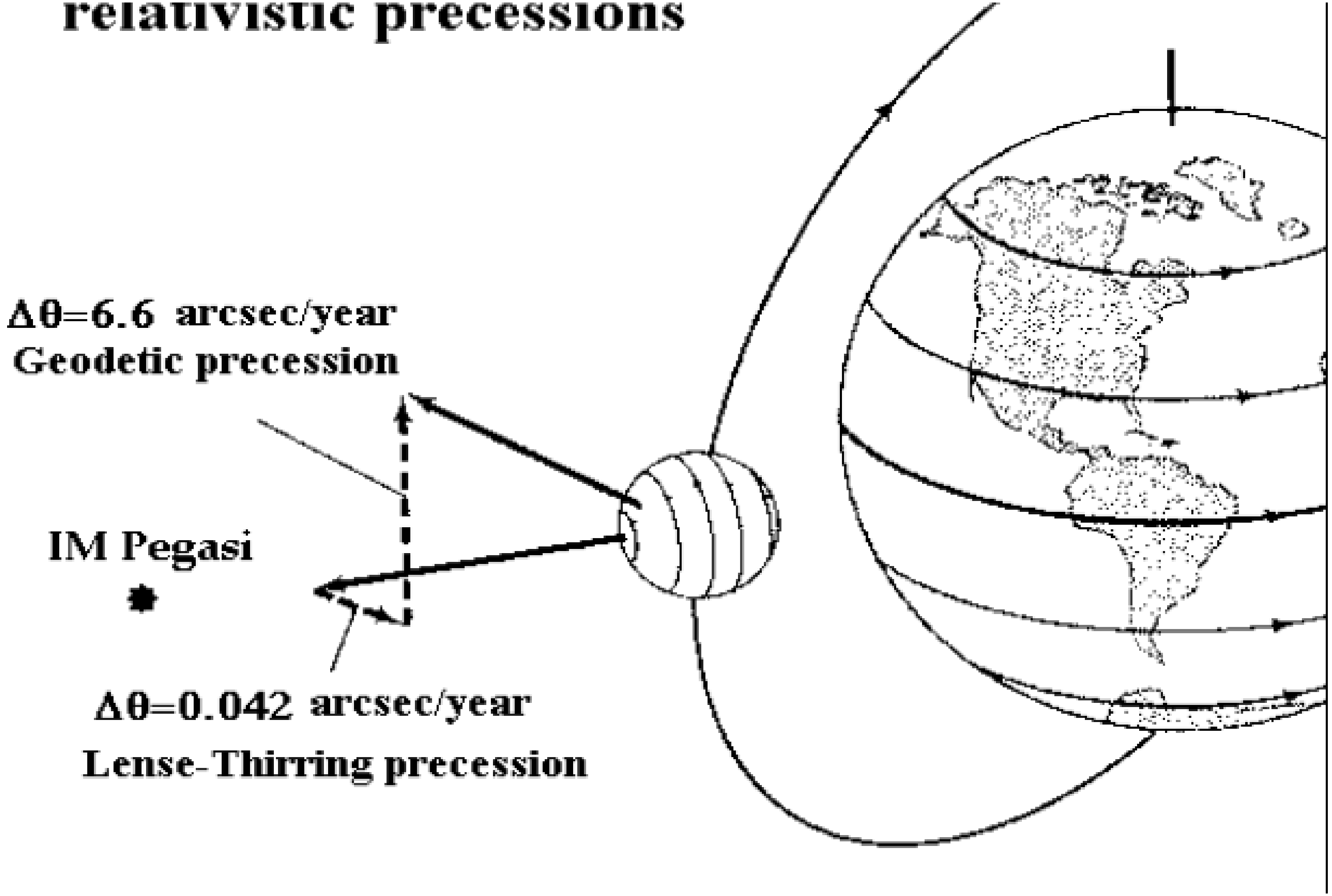


z
B
y
S
Ω
x



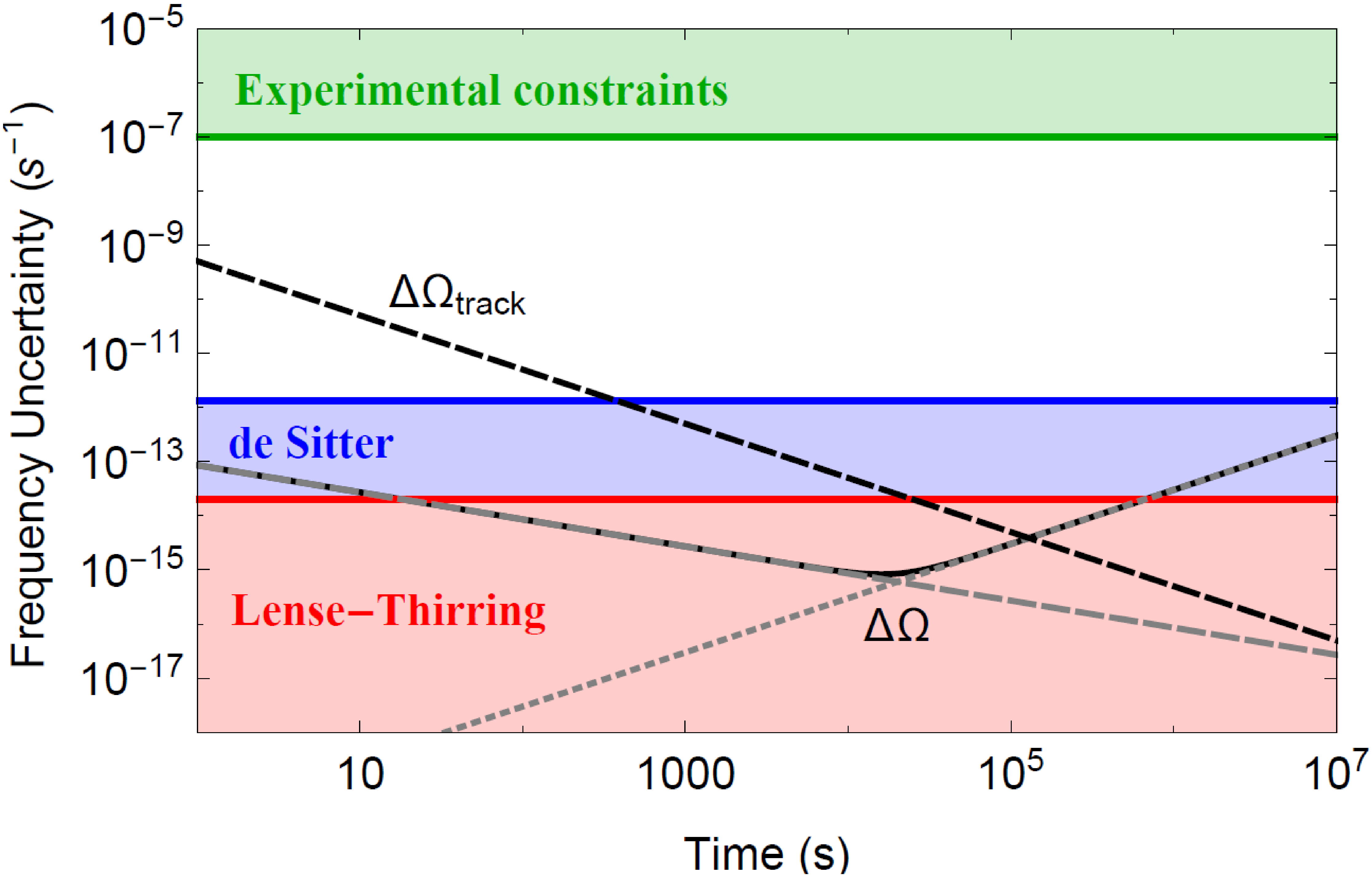
Classical mechanics
Electro-magnetism
Quantum mechanics
Newtonian gravity
Special relativity
General relativity
geodesic motion
Einstein's equations
Quantum field theory
Quantum mechanics on curved space-time
QFT on curved space-time
Linearized quantum gravity
Quantum gravity phenomenology
Quantum theory of gravity

relativistic precessions

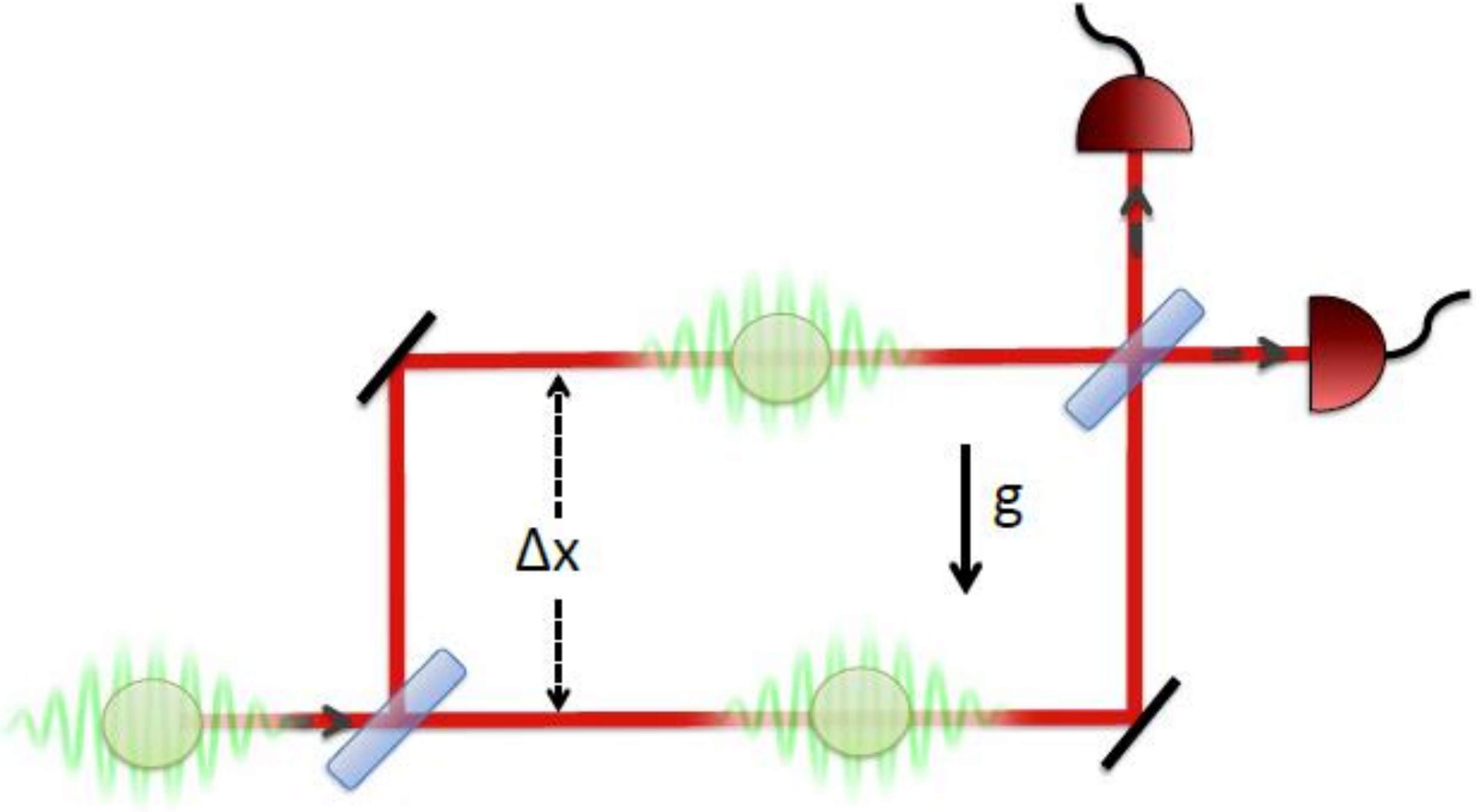


Newtonian vs General Relativistic gravity

Newtonian field equations	GR field equations
$\nabla^2 \Phi = 4\pi G \rho$	$G^{ab} = \frac{8\pi G}{c^4} T^{ab}$
Source: mass density Gravitational field: scalar Φ	Source: energy-momentum tensor (includes mass densities/currents) Gravitational field: metric tensor g_{ab}



Experimental constraints
$\Delta\Omega_{track}$
de Sitter
Lense–Thirring
$\Delta\Omega$
Frequency Uncertainty (s^{-1})
Time (s)
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10
1000
10^{5}
10^{7}


Δx
g